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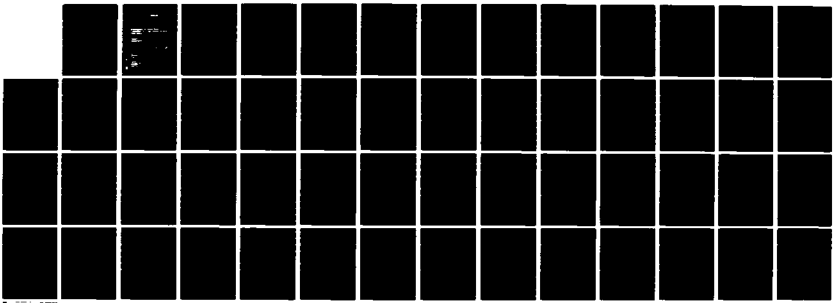

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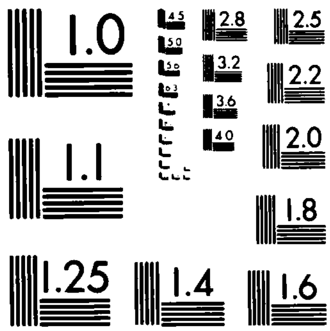

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INVESTIGATION OF WAVE ROTOR TURBOFANS FOR CRUISE MISSILE ENGINES

FINAL REPORT

Volume 2
APPENDICES

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Appendix A

WAVE ROTORS FOR AIRCRAFT TURBINE ENGINES

ABSTRACT

The history of wave rotor applications to aircraft turbine engines is reviewed in order to determine the reasons for past successes and failures. The results show that two different kinds of wave rotors performed successfully in laboratory tests in the mid-1950s and late 1960s. These were the Pearson wave rotor/turbine built and tested by Ruston Hornsby in England and the Comprex pressure exchange wave rotor built by the Brown-Boveri Company and tested by the Rolls-Royce Company.

With the advent of advanced gasdynamic numerical simulation techniques embodied in an experimentally verified computer flow code for wave rotors, the prospect of upgrading these earlier wave rotor designs for a modern high temperature, high pressure aircraft turbofan engine looks very attractive. The flow code would allow very rapid design optimization and cut down the time and costs for successful engine development. A suitable niche for the initial application of wave rotor turbofans has appeared in the area of small, low-TSFC engines; wave rotors may also allow better cruise conditions for high performance engines.

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INTRODUCTION

Wave rotor turbo-fan engines appear to have the potential for achieving significant gains in thrust-specific fuel consumption, specific power, acceleration, and reliability. However, one must ask why no successful aircraft engines using wave rotors emerged from the 1950-1970 time period when they were being investigated most intensively for that application.

Several companies participated in wave rotor research, including General Electric, Rolls-Royce, Brown-Boveri, and smaller companies such as Power Jets Ltd., ITE Circuit Breakers, the General Power Corporation, and Ruston-Hornsby. Of these, only Brown-Boveri has carried out a successful commercial wave rotor development program for the Comprex™, which is used as a supercharger for diesel engines.

The purpose of this document is to sift through the history of wave rotor research to determine what went right or wrong and why, and to use this information to identify those wave rotor approaches which make the most sense for aircraft turbine engines. This review shows that two wave rotor turbine engines achieved limited but significant successes in laboratory tests; also, that these two research programs were cancelled primarily for economic, not technical, reasons. The first of these, the Pearson rotor, was built and tested by Ruston-Hornsby in the mid-1950s and produced net shaft power from the moment it was turned on. It was taken through hundreds of hours of tests over a wide range of operating conditions before being terminated during an economic downturn. The second, the Rolls-Royce tests of a Comprex pressure exchange wave rotor for turbine engines, is less well documented but roughly equal in success to the Pearson rotor by virtue of achieving a predicted level of performance for a wide range of operating conditions after several years of test stand evaluation. The Rolls-Royce research program on wave rotors was terminated when the company went into receivership in the early 1970s.

There are two main reasons for using wave rotors for aircraft engines. The historical reason - a relatively high component efficiency in a device that is automatically cooled at high temperatures - has been partly borne out in practice; namely, the high temperature capability was realized with the materials available at that time, but high component efficiency was harder to demonstrate because of the lack of design optimization techniques and leakage control. The second reason, which has not been recognized until recently, is the capability of very fast wave rotor response to external throttle conditions, allowing a wave rotor turbine engine to operate much closer to the compressor stall line. This attribute means that highly maneuverable engines can be upgraded for higher performance cruise conditions, and weapon reliability can be increased against inadvertant or sudden pressure fluctuations in the engine (eg., at start-up or transition to dash).

Recent advances make it worthwhile to reconsider the use of wave rotors for aircraft turbine engines at this time. These advances are the recent improvements in computational fluid dynamics, the acquisition of critical new experimental data on wave rotor performance, and the fact that advanced turbine engine performance is constrained more than ever by high temperature materials development. A computer flow simulation code has been developed which allows very accurate prediction of wave rotor performance verified by detailed data taken at the MSNW wave rotor test facility. This code has been used to rapidly modify the original wave rotor designs for higher efficiency and to measure that efficiency increase in subsequent wave rotor tests. The capabilities of this vastly improved computational technique overcome one of the primary causes of failure (i.e., slow, burdensome design optimization) in earlier efforts and should make it possible to design and develop a successful wave rotor engine for a reasonable cost in a relatively short period of time.

In summary, historical evidence supports at least two important examples of successful wave rotor operation in the context of aircraft turbine engine applicatons; these instances were as successful as they

could have been with the limited tools available to the designers at the time. The Comprex development by Brown-Boveri has established the fabrication techniques for wave rotors in commercial quantities, and the MSNW experimental program has established the possibility of optimizing wave rotor design very quickly for high component efficiency by the use of modern computational tools. With the recent successful development of fast computational design codes for wave rotors, it is time to look again at applying these devices to those aircraft applications where they have their greatest advantage; namely, to small, low cross section, long-range engines, and to high maneuverability engines. In short, wave rotor technology has arrived at that point in its development where it is ripe for aircraft engine application.

The Department of Defense is currently supporting a limited range of wave rotor experiments. To a certain extent these tests are tracing over ground already demonstrated successfully by the Pearson rotor in 1954. To advance beyond what was learned 28 years ago, a more sophisticated wave rotor design must be developed for higher temperature operation, building on the Ruston-Hornsby and Rolls-Royce successes, and that design should be optimized using the latest techniques and data. The resulting device should be subjected to well-instrumented tests to verify its performance and to evaluate control of seals, clearances, and leakages. If these results are positive, then it will be possible to proceed confidently with an engine development program.

SUCCESSES AND FAILURES

Wave rotor turbine engine research was concentrated in the mid-1950s to late 1960s. During this period, aircraft turbine engines rose to prominence, achieving large advances in component efficiency and reliability. This feat was accomplished by metallurgical advances and by design improvements which reduced the aerodynamic losses. Those advances were made possible by designs which treated the turbines and compressors as steady flow systems, albeit with some important transient effects, such

as compressor stall. The steady flow assumption vastly simplified the design problems and allowed much of the progress that occurred at that time.

The wave rotor, also in its infancy in the early 1950s, is patently an unsteady flow device. Consequently, it was much more difficult to understand and to design. Similar to other new technologies, there were a multitude of wave rotor configurations under consideration at that time, which compounded the problem of deciding which approach was the most appropriate for any individual application.

In retrospect, the historical choice to favor axial flow turbine and compressor research at the expense of wave rotor research seems obvious because the payoff was high and the perceived costs, in terms of relative design simplicity, were lower. It is difficult to compare these two technologies since many of the companies previously involved in wave rotor research still are hesitant to discuss their decisions regarding the cessation of that research. Nevertheless, it is possible to arrive at a convincing group of reasons for what happened by examining several concrete examples.

THE KLAPPROTH ROTOR

The General Electric (GE) experience is possibly one of the best documented examples of corporate wave rotor research. Over a period of 8 to 10 years, GE considered several wave rotor configurations in the context of advanced turbine engines. This predated the most intense interest in turbofan engines by the aircraft industry, so that the engines considered were turbo-props. They carried out an extensive experimental and analytical program which involved a number of engineers and scientists. Both pure pressure exchange and shaft work output wave rotors were investigated. The former took the form of a device very similar to the modern Comprex device (a commercial wave rotor presently sold by the Brown-Boveri Company as a diesel engine supercharger); it had straight

tubes parallel to the rotor axle. The second device, which we have dubbed the Klapproth rotor since it survives to this day and was associated with Klapproth's efforts in the GE research program, has helical tubes which are capable of receiving impulsive thrust from the inlet gas flows and which can deliver reactive thrust from exit gas azimuthal velocities different from the rotor tip speed.

Though the record is silent on this point, it appears that each of these rotors succeeded to a certain extent; namely, to validate the compression and expansion processes occurring with each cycle on the rotor. But the Klapproth rotor in particular apparently never succeeded in producing significant shaft work output. In this mode it would have operated as a compressor-turbine combination with a combustor to add energy to the compressed air flow. Conversations with some of the principals involved indicate that this device could only sustain its own rotation, but no net work ouput of any reasonable amount (i.e., close to theoretical projections of work output) was measured. One might surmise that its component efficiency was low.

At that time it took a gas dynamicist many weeks to generate a wave diagram describing the internal gas flows for a wave rotor. This was accomplished via painstaking hand calculations using the method of characteristics. Even the smallest design change would require a recalculation, with a several week delay in information while the new flows were calculated. These calculations were supplemented by some excellent and complex water table experiments which could simulate some of the time-dependent and design-dependent aspects of wave rotors and their interactions with the stationary supply and exhaust manifolds. However, there was a basic lack of empirical data and a lack of understanding of how these devices actually behaved because of inadequate modeling.

For example, throughout this period (and even to this day) there has been a temptation to describe wave rotor operation with the use of a simplified representation of the wave processes. This involves the

initial assumption that the waves could be drawn with zero width, as if they were generated at a discrete point in time and space and did not spread or deform later in time. Further, little or no recognition was given to the need to account for at least three internal reflections of these waves in order to properly calculate the flow magnitudes. Possibly the most important requirement of steady flows in the manifolds external to the wave rotors appeared to be assumed without also requiring that the wave patterns used in the design be periodic with each revolution. As a result of this last assumption, the design calculations generally mis-estimated the actual performance by a substantial amount.

This retrospective appraisal of the GE effort has been made possible by recent research on the part of Mathematical Sciences Northwest Inc. (MSNW) under a program supported by the Department of Energy and under a current analytical effort supported by DARPA.

When each of these design requirements is taken into account, there is good reason why the Klapproth rotor and others like it did not always perform satisfactorily. With the aid of modern computational techniques, we can improve the design of the Klapproth rotor and identify those applications where it makes the most sense.

The lesson is that new design restraints have been recognized which will allow an improved design. Wave rotors can now be analyzed and designed accurately and quickly. Such analysis is now supported with a limited data base. This data base could be quickly expanded with a few well-chosen experiments in order to verify the potential of the device before entering into a more ambitious development program.

THE ROLLS-ROYCE ROTOR

In this instance, the available information is sparse. At present, we know that Rolls-Royce (RR) carried on an experimental test program and a parallel analytical effort. Consultants to Brown-Boveri assisted RR in

this latter effort, carrying out some of the numerical calculations required to generate the wave diagrams and rotor designs. As in the GE case, these calculations were performed by hand and each design took many weeks to complete. The RR program began in the 1960s and perhaps lasted as late as 1972, when the company went into receivership and was taken over by the government. Clearly, the loss of revenue at that point stopped numerous research efforts, many of them successful, and some of them relatively high risk, such as the composite blade program for the Lockheed L1011. The wave rotor program was no exception; however, it is known that they were successful in reaching to within 80 percent of their design goals, but the program was cancelled along with all the other research in 1972.

So far as we can tell, the Rolls-Royce rotor(s) were pressure exchange wave rotors; in fact, at least one of them was simply a Brown-Boveri Comprex with the same manifolds (though different placement) as those used with the diesel supercharger. The application appears to have been to a gas turbine cycle with a wave rotor high pressure stage as a topping cycle, as described by Berchtold and Lutz in their papers. Test data of a limited sort was evidently taken to try to verify the performance of this device. This data apparently included mass and energy flows for each of the manifolds and some measure of the gas leakage, which was difficult to control and, from their viewpoint, precluded the achievement of the pressure ratio they were looking for. In a later development, they were able to reconcile the predicted performance with measured data and could consider that their program had met their objectives.

Since that time, RR does not appear to have resumed their research program, choosing instead to concentrate on areas having major payoffs for their larger engines, such as transonic propellers. However, there is no fundamental reason why they might not choose to return to wave rotors in the future, if they see a productive application for them.

To conclude, the RR experience was much better than GE's since their device operated according to their prediction. However, they suffered some of the same problems regarding the difficulty in understanding the gas flows and in developing optimized designs for their wave rotors. They were also able to identify leakage as a significant mechanical design problem in their experiments.

It would be very valuable to be able to scrutinize more of the RR wave rotor experience. Any details on the actual performance measured and their assessment of technology problems would add to our understanding of what needs to be done to make a successful wave rotor. In balance, however, they achieved definite, positive performance results. What problems they did have could have been alleviated by the availability of a more rapid design tool plus a better conception of what constitutes the most efficient design.

THE PEARSON ROTOR

In the mid-1950s, the Ruston-Hornsby Turbine Company supported the construction and testing of a different kind of wave rotor designed by Ronald Pearson. The blades of this device had long helical sections and cambered ends. That is, the blades or tube walls were bent somewhat at each end, which changed the direction of the gas flows more like a conventional turbine blade. However, the Pearson rotor was very much a wave rotor since part of the cycle was devoted to compressing the inlet air while the rest of the rotor cycle utilized the expansion of the exhausting gas (heated by an external combustor) to extract shaft work.

Based on experience with a smaller prototype rotor, the Pearson wave engine was designed and built in less than a year, and it produced shaft power from the first moment it was fired up. In this regard, the Pearson rotor was singularly successful. It operated over a relatively wide range of off-design conditions, producing net shaft work in the range of 5 to 35 horsepower, which was very close to its projected operation.

While Pearson also had to perform many tedious wave diagram calculations by hand, he had adopted a more modern design philosophy from the start. First, he recognized that the wave system must be periodic and enforced that condition in all of his calculations. Second, he accounted for all of the internal wave reflections and, in many instances, designed extra ports whose purpose it was to control and/or cancel these reflected waves. He also took a fundamental approach to guarantee that his wave diagrams would produce net work output; that is, the pressure, mass flow, and temperature conditions imposed as boundary conditions on the wave diagram calculations were prescribed in such a way as to produce net work. This approach is not evident in any of the other wave rotor designs which we have encountered, yet it is self-evident that it is a requirement for a constructive approach to the design process.

The Pearson rotor was also very advanced for its time in terms of using abradable seals which could be held to close clearances and in the brazed construction of the rather convoluted rotor tubes. The bearings required special attention, and the thermal expansion of the rotor components was very carefully considered.

The wave rotor experiments were clearly outside the norm for projects at Ruston-Hornsby and were the brainchild of one of the directors of the company who took a special interest in Pearson's device. As a result, the project was considered rather expendable and, when it suffered a setback due to overspeeding from an improperly connected fuel line, the project was canceled and Ruston returned to its customary product development. This also coincided with economic bad times in the company and can be viewed as a natural attrition of extracurricular activities occurring at such times.

The Pearson rotor is a very important piece of evidence supporting the idea that wave rotors can be made to work effectively. Its success refutes the claim that such devices only work over a very narrow operating range. The Pearson design contains specific ports and nozzle vanes whose

sole purpose it is to maintain high component efficiency over a very wide range of operating conditions. These design details were tested and improved upon during the project at Ruston-Hornsby; the experimental data supported the worth of each of these remedies. The only other rotor including the capability for good off-design performance is the Brown-Boveri Comprex, which positions closed cavities or "pockets" at points around the periphery of the end walls in order to control wave reflections at those walls. Again, the Comprex experience and data also support the capability of wave rotors acting as pure pressure exchangers to operate over a wide range of conditions, especially at part load.

THE GPC ROTOR

The General Power Corporation (GPC) is one of the few firms to have maintained some degree of continuity in a wave rotor research program, having begun in the mid-1960s and still being active now. The GPC rotor shares some of the features of the Klapproth rotor and of the Pearson rotor, but there the resemblance ceases once one considers the details of its design and operation. Its purpose is to generate shaft horsepower much like the Pearson rotor, and it utilizes helical tubes with a bend (camber) near the outlet to change the direction of the gas flow on the rotor. The main part of each tube has a stagger angle much like the Klapproth rotor (i.e., helical blades). In the GPC patent disclosures, the wave diagrams were described with infinitesimally thin waves, with no spreading in time. Thus, one might suspect that the designers were ignoring some of the fundamental design constraints required for a successful machine.

The GPC rotor development was originally intended for a road vehicle engine, which would require a relatively high pressure ratio Brayton cycle with several expansion stages on the wave rotor. The curved blades were intended to act as a reaction turbine for this portion of the flow in order to produce net shaft output power. Each stage was fed by the preceding exhaust stage gas with a re-entrant duct which curled around

from one side of the rotor to the other. Work on this device has been sporadic, being supported first by the Ford Motor Company and later by the Department of Energy.

A simplified version of the GPC rotor, which includes just one re-entrant duct for a lower overall pressure ratio, is being tested under support by DARPA. However, the basic rotor design has not changed significantly. To date, this rotor has not produced any net output power. An initial inspection of its design suggests that the tube curvature may be too large; that is, too much reaction may be encountered during the expansion stages, robbing the exhaust flows of the pressure they need to re-inject the flow into the next expansion stage. Hence, one would suspect poor scavenging under these conditions with a subsequent loss of available output power.

Unfortunately, the GPC work is poorly documented at present, making it difficult to draw any certain conclusions about the reasons for its apparent failure. When contrasted to the Pearson device, there are several areas of the GPC design which are suspect. The first, discussed above, concerns the degree of bending in the tubes. The second concerns the lack of control over reflected waves within the device required to make the wave system periodic within one revolution. The third area is the absence of any strong impulsive loading of the rotor from inlet manifolds to produce shaft work; the Pearson device relies heavily on impulsive loading to achieve power output and has only a very mild change in the blade angle to produce a small amount of reactive power.

Any one of these design problems would be sufficient to cause severe problems in power output from a wave rotor/turbine. The GPC difficulties were compounded in the earlier years by the same difficulties in computing the wave patterns which other researchers had. They have recently developed a characteristics computer code which is potentially accurate and fast but involves a very complex accounting system of characteristics nodes in order to achieve high accuracy; each time a wave reflects within

the system, it adds to the number of such nodes which must be accounted for and multiplies the length of time that such a computation must run to get good results. Even so, such a technique will cut many hours from the normal design calculation done by hand.

The GPC approach began with an enormously complicated rotor design that was destined to be a complete vehicle engine and so they were initially encumbered with a multitude of design problems which were somewhat extraneous to the task of learning how the wave processes would evolve on their device. Some of those design problems have been solved, but the GPC rotor is still not a good system for learning about or verifying wave rotor performance.

Our preliminary assessment of the present GPC device is, as described above, that it is deficient in several important respects and may never produce net output work. It would be absolutely essential to get very good measurements from such a device if it were the only possible approach to the wave rotor/turbine. Fortunately, the Pearson rotor is an alternate which works and works well, so it may be completely unnecessary to pursue the question of the GPC device performance further at this time.

THE COMPREX

The Brown-Boveri Company has been involved with wave rotors since the earliest period in its history and, in the beginning, their staff and consultants tried a number of different applications including the use of wave rotors as topping stages in a turbine engine. Ultimately, they settled on the development of the Comprex™ as offering the maximum corporate payoff consistent with their corporate objective. The Comprex is a supercharger for diesel engines. This device has tubes parallel to the rotor axle and is designed to have a uniform performance over a wide range of road conditions. The Comprex is a commercial device and has been successfully tested on a wide variety of vehicles such as the Mercedes-Benz diesel car, the Peugeot, and Ferrari, on diesel trucks

manufactured by M.A.N., and on bulldozers. A great deal of development has been applied to making the Comprex a durable device which meets the stringent conditions of modern automobile engines; it currently is sold as the factory installed supercharger on a class of European-produced tractors.

The Comprex does not have to be very efficient in the supercharger application because there is generally surplus available work in the engine exhaust stream over and above that needed to compress the incoming air (i.e., to supercharge the engine). At low part load, any deficiency in available work is rapidly compensated for by the ability of the Comprex to rise quickly to full load performance. Ordinary turbochargers suffer some lag in this respect because of their rotational inertia. As a result, the current Comprex designs emphasize a broad performance range instead of high efficiency. As mentioned earlier, pockets are cast into the rotor endwalls to control wave reflections and the internal gas flows to achieve this broad performance, but the Comprex does not attempt all of the refinements needed to also achieve high efficiency; for example, those found in the Pearson device.

The Comprex wave rotor is one of the most important examples of successful wave rotor technology. It is a commercial device. Furthermore, the Brown-Boveri Company has solved the most difficult development problems for their application - the seals problem and the thermal stress problem. Leakage is kept to an acceptable level in the Comprex by enclosing the rotor in a pressurized shroud and by using a rotor made of a special alloy having a low thermal expansion coefficient over the range of temperatures suitable to the Comprex. The thermal stresses are managed by routing the hot gases in and out the same end of the rotor and the cold gases in and out of the other end. Also, a "cathedral" design using an alternating arch cross-sectional shape for the tubes helps to allow thermal growth in the tube walls without placing large stresses on the outer rim of the rotor.

Through conversations with the Comprex staff, it appears that the Brown-Boveri management views the supercharger application as having the greatest corporate payoff at the present. Their earlier problems in achieving high performance in the turbine engine application came at a time when gas turbines were advancing rapidly, as described earlier in this paper. Many of the same personalities working on those problems then are still at Brown-Boveri. These same individuals have evidently regarded the risk of re-entering the turbine engine application too large for the expected commercial payoff at this time.

The foregoing assessment of corporate risk and the role it has played in wave rotor development decisions is critical when viewed from a strategic point of view. Corporate profit has not been sufficient for aircraft engine companies to carry out certain aspects of advanced development by themselves. For example, were it not for government support of military aircraft engine development, such engines would also probably not have been built. Similarly, it may be worthwhile for the government to assume somewhat greater risk by underwriting the initial stages of rotor development in order to ensure that a strategic goal is met.

HISTORICAL SUMMARY

Of the multitude of wave rotors tried in the past, two have been successful in terms of approximating closely their projected design performance. These were the Pearson rotor and the Rolls-Royce rotor (alias the Brown-Boveri Comprex). Their success appears to be due to a concerted effort to understand the internal wave processes and to incorporate wave control remedies in the rotor design. These remedies were also aimed at providing a very wide range of operating conditions for these rotors without undue fall-off in performance. The Pearson device, in particular, achieved net work output by also understanding the fundamental trade-off between reactive forces and impulsive forces and the need for closed loop gas scavenging on the rotor.

Much more could be said about the details which led to respective success and failure in this area. We have mentioned the interplay also between the rise and fall of the business cycle and competing technologies during this period of time. A new set of conditions has emerged in the past ten years which provides a strong incentive to reconsider wave rotor technology. This is the fact that high performance turbine engines are now reaching further into the regime of difficult-to-develop, high temperature materials. This incentive is discussed later with the idea of motivating a discrete range of application of wave rotors; namely, to small turbine engines where blade cooling of conventional turbines blades becomes burdensome, and where high component efficency is hard to obtain. But first it is worthwhile to summarize the most recent development of new tools for optimizing the design of wave rotors.

RECENT WAVE ROTOR DEVELOPMENTS

MSNW completed a 3-year wave rotor technology program in 1981. As a result of that effort, critical new experimental data on wave rotor performance were acquired that demonstrated the capability to design and predict the performance of high efficiency wave rotors in detail. These experiments were conducted at moderate temperatures and pressures in order to utilize accurate, reliable instrumentation for gathering the data. Work transfer efficiencies of 74 percent were measured in a small unoptimized (100 kW) device; this corresponds to the product of a compressor times a turbine efficiency (e.g., 0.83 X 0.89). At the same time, these results were corroborated in detail by correlating the data taken from on-rotor pressure transducers with the output from a computer flow code calculation. The flow code uses the SHASTA flux-connected transport algorithm for integrating the one-dimensional Euler equations of unsteady gas dynamics. The code incorporates heat transfer to and from the rotor walls, nonuniform flow into the manifolds, leakage at the rotor seals, viscous drag along the rotor tube walls, and a host of other real-device effects.

This code also has been used successfully to predict design changes for improved performance, which were then tried and confirmed experimentally. The procedure for re-design and code prediction is very rapid so that many possibilities can be considered computationally to optimize performance of a design for a particular application before constructing and testing the device. The MSNW experiments validating this process lends a high degree of confidence that this technique can be extended to other rotor designs and applications. The current DARPA-funded program at MSNW is aimed at carrying out code extensions to wave rotor turbines and to pressure exchange wave rotors for high temperature, high pressure ratio aircraft engine applications. The preliminary results from this program suggest that this technique should also be capable of yielding accurate results for more complex wave rotors.

WAVE ROTOR TURBOFANS FOR THE 1980s

As we have seen, the past history of wave rotor development is peppered with mixed levels of achievement. We think we understand the reasons for this and now are in a position to give a realistic appraisal of wave rotor technology as a way to improve the performance of present-day turbine engines.

During the last ten years, the limit to turbine engine performance has been measured in part by the peak turbine inlet temperatures allowed by turbine blade materials and by blade cooling. Blade cooling puts an extra drain on the compressor and begins to reach a point of diminishing returns when the temperature difference between the inlet and the blades increases to a few hundred degrees. Thus, the burden for further improvement, at least in the core engine, falls back on high temperature materials development. Significant improvements have been made over the past decade on hot stage turbine materials; reliable performance can now be achieved with inlet temperatures of 1900 to 2000 F, and for limited

durations up to as high as 2500 F without blade cooling. With blade cooling, these limits can be extended upwards by a few hundred degrees.

The wave rotor automatically cools itself without diverting any gas streams from the compression part of the cycle. Because the cooler input air is periodically cycled on and off of the rotor, the rotor wall sees both the cold and the hot gas streams and assumes an intermediate temperature between these two. The actual rotor wall temperature depends on the details of the heat transfer with these two gas streams but is generally much lower than the inlet combustion gas temperature. Further, no special passages have to be drilled into the walls of the wave rotor in order for it to be cooled. The cooling is accomplished within the main gas passages consisting of the compression and expansion tubes surrounding the rotor. As a consequence, cooling in small wave rotors is achieved with ease compared to the problems of blade cooling in small turbines. Since the wall temperature is somewhere near the mean of all of the inlet and outlet gas temperatures, the rotor temperature may differ by considerably more than a few hundred degrees from the peak gas temperature, implying that higher peak gas temperatures may be used with this device than with a cooled gas turbine. Of course, any material improvements available for advanced turbines may also be used to boost the temperature capabilities of the wave rotor. In essence, the wave rotor can be considered as a way to save the turbine from high inlet temperatures.

We submit, therefore, that the wave rotor should be reconsidered, specifically as a high temperature, high pressure stage in a turbine engine because it will protect the power extraction turbine from the peak gas temperatures of the combustor. Its high temperature capabilities should be used to boost the cycle efficiency (and lower the thrust-specific fuel consumption) of present-day gas turbine engines and to relieve some of the necessity of entering a difficult materials development program for the more advanced turbine engines. By keeping the high tip speed turbine components at a lower temperature, the wave rotor

will also improve the overall reliability of these engines; this is an important consideration whether for a dependable weapon system or for a safe commercial application.

The chief question underlying these suggestions to reconsider the wave rotor is no longer whether or not it will work; we now have sound historical evidence that wave rotors can and will work. Rather, one needs to demonstrate how well these devices will work, since their component efficiencies at higher temperatures will be compromised by the amount of heat transfer and gas leakage which accompanies high temperature applications. Some of these concerns are being addressed analytically by the current DARPA program at MSNW, where a detailed flow code is being used to evaluate heat transfer, leakage, and component efficiency effects for a variety of wave rotors. The initial estimates of these effects indicated that the component efficiency can be maintained at a high value. Also, since the wave rotor is being considered primarily as the high pressure stage, the overall engine performance is not particularly sensitive to the efficiency of that stage; that is, any losses from work transfer or shaft work production in the top stage are available to do work in subsequent stages, although not with the same effectiveness as if that work were accomplished in the top stage. This cascade effect is well documented in ground-based, combined cycle operation, where the topping cycle is often rather inefficient but chosen simply for its ability to operate at higher temperatures (for example, the MHD-steam turbine combined cycle power system) and to thereby boost the overall thermodynamic cycle efficiency.

Besides its advantages of cool walls and good efficiency in small sizes, wave rotors appear to behave in a substantially different way to pressure transients in an engine configuration, compared to a conventional turbine engine. One of the principal limits to high performance aircraft engines is the surge line marking the beginning of compressor stall. If the throttle is opened too quickly by the engine operator, the pressure rises in the combustor and creates a temporary decrease in the equivalent

mass flow exiting from the compressor. Because of the inertial masses of the compressor-turbine pair, the compressor maintains roughly constant rotational speed during this transient, moving the compressor performance across the surge line where it will stall, with an immediate loss in power. Thus, the operator must not accelerate the engine too rapidly. Military engines are derated below their performance potential so that an adequate safety margin is provided for acceleration without compressor stall.

The transient response of the wave rotor is extremely fast, since the wave patterns internal to the device can readjust on time scales like the acoustic transit time along the length of the rotor, e.g., on the order of a millisecond. The rotational speed of the rotor is not the primary variable in this instance since it controls the valving of the gas on and off of the rotor but not the mass flow or pressure constraints of the system. As a result of its fast response time and because it is placed between the combustor and the compressor, a wave rotor will allow a turbine engine to operate much closer to the compressor surge line without any danger of compressor stall. The implications of this change are that the overall performance of the turbine engine can be increased under ordinary flight conditions, and it will be effectively stable to either accidental or intentional changes in the combustor (i.e., throttle) conditions on very short time scales. Thus, for weapon systems, a wave rotor turbine engine can be very reliable since it will not accidentally stall and it can be programmed or driven to very rapid maneuvers. Similarly, in commercial aircraft, such an engine can be operated under cruise conditions closer to the stall line and hence at higher efficiency and lower thrust-specific fuel consumption to obtain increases in range; for unanticipated situations (an aborted landing, for example) the pilot would have the capability of pulling out (i.e., accelerating) with a much smaller margin than with a conventional engine.

The transient response of the wave rotor has been demonstrated during the test series performed by Ruston-Hornsby on the Pearson rotor. There

is every reason to believe that this is a general property of such devices but, of course, it should and can be confirmed by analysis and experiments for any given wave rotor design.

CONCLUSIONS

The history of wave rotor turbine engines shows that two such engines have been built and demonstrated in the lab close to their projected performance over a wide range of operating conditions. These are the Pearson wave rotor/turbine and the Rolls-Royce (or Comprex) pressure exchanger wave rotor. New computer design techniques verified by recent experimental data now allow rapid design optimization of wave rotors so that a successful development program for these devices could be completed in a relatively short period of time.

The development of advanced turbine engines is pushing the limits of high temperature materials. Wave rotors would boost the high temperature limits to turbines, allowing better and more reliable performance, especially in small-size engines where cooling and component efficiency are difficult to achieve. No new materials development would be required for the wave rotor turbine engine to yield significant improvement over existing turbines. If higher temperature materials become available in the future, they would simply increase these improvements.

Both larger and smaller wave rotor turbine engines would benefit from the wave rotor's ability to stabilize engine transients during sudden changes in the throttle conditions, allowing better cruise performance, maneuverability, and reliability.

The government stands to benefit by taking the risk of initial wave rotor development. Specifically, the aircraft engine industry is pursuing advances in commercial aircraft engines which show payoff at moderate risk, but these approaches do not necessarily address the Defense

Department's needs, especially in the two areas of low cross section, long range small engines and high maneuverability engines.

The government is pursuing wave rotor research on two fronts: an analysis of possible wave rotors to determine the best approach for small aircraft engines and an experimental effort to measure net work output from a specific, existing wave rotor/turbine. A critical need exists at this point which is not being addressed by either of these two project areas; namely, a detailed experimental verification of the performance projected for hot stage versions of either the wave rotor/turbine or the pressure exchange wave rotor, or preferably both. To a certain extent, the current experimental program is tracing over old ground; namely, to demonstrate again what the Pearson rotor has already demonstrated in 1954.

To advance beyond what Pearson and Ruston-Hornsby learned 25 years ago, it is necessary to incorporate improvements in the Pearson rotor and to run well-instrumented tests on its performance in the design range of operating conditions required for a particular application. The chief requirements in this regard are to measure and control the leakage and to manage the design for a higher peak temperature than was considered at that time. The knowledge gained from the DOE and DARPA wave rotor programs at MSNW has provided the basis for the design of a wave rotor with the highest possible component efficiency.

While the case discussed above was a wave rotor/turbine, the pressure exchange wave rotor may offer a lower risk, faster development alternative. The pressure exchanger version of the wave rotor typically does not have to operate at as high a tip speed as the wave rotor/turbine because it does not have to meet propulsive efficiency requirements associated with the high speed of sound of the combustion gases. It may also be possible to make a more efficient design for the pressure exchanger. The combination of lower tip speed and higher efficiency means it would be easier to develop a highly reliable wave rotor for a given

peak cycle temperature. There may be some penalties to this approach in the complexity of the resulting engine since an extra, rotating component beyond the turbine and compressor is required. A careful design of the turbine engine configurations resulting from these two wave rotor approaches needs to be carried out to answer this question.

In short, the potential for wave rotor turbofan engines is attractive. Wave rotors have been operated successfully in the laboratory in the past. Those designs can be improved upon quickly to upgrade their efficiency and performance. Well-instrumented tests of a wave rotor at higher temperatures and pressures would establish the feasibility of this technology and confirm the validity of the existing design tools which would be needed for an engine development program.

Appendix B

SUMMARY OF ON-DESIGN AND OFF-DESIGN EQUATIONS

ON-DESIGN EQUATIONS

Inputs (Refer to Figure B-1 for nomenclature)

T_o °R	γ_{4L}	γ_4	γ_9	$\gamma_{9'}$
γ_{5ML}	h(BTU/lbm)	π_d	π_{BL}	π_{BH}
π_M	π_n	$\pi_{n'}$	η_{BL}	η_{BH}
η_{AB}	$\eta_{AB'}$	$e_{c'}$	e_c	e_{EC}
e_{tM}	e_{tL}	e_{Et}	P_g/P_o	$P_{g'}/P_o$
$\tau_{\lambda L}$	τ_λ	$\tau_{\lambda AB}$	$\tau_{\lambda AB'}$	$\pi_{c'}$
π_c	π_{EC}	M_o	α	

Equations

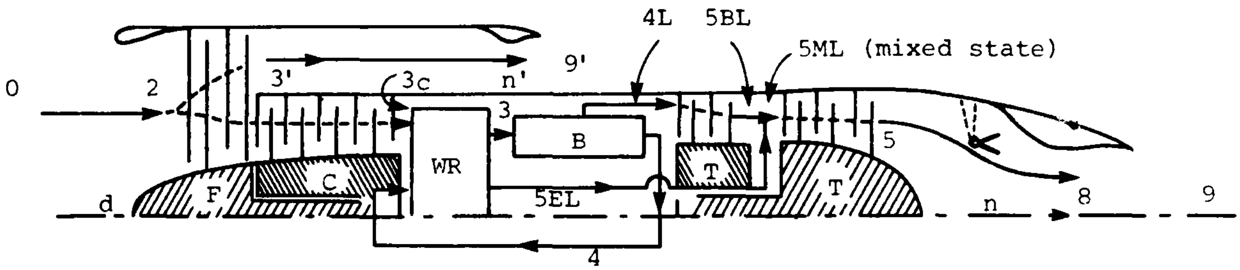
$$a_o = 49\sqrt{T_o}$$

$$\tau_r = 1 + \frac{M_o^2}{5}$$

$$\pi_r = \tau_r^{3\,5}$$

$$\tau_c = \pi_c^{\frac{1}{3.5\ e_c}}$$

$$\tau_{c'} = \pi_{c'}^{\frac{1}{3.5\ e_{c'}}}$$



f_4, f_{4L} = fuel-to-air ratios, ϵ = mass flow fraction at 4L

$$\tau_r = \frac{T_{t0}}{T_o} = \frac{T_{t2}}{T_o}, \quad \pi_r = \frac{P_{t0}}{P_o}, \quad \tau_d = \frac{T_{t2}}{T_{t0}} = 1, \quad \pi_d = \frac{P_{t2}}{P_{t0}}$$

$$\tau_{c'} = \frac{T_{t3'}}{T_{t2}}, \quad \pi_{c'} = \frac{P_{t3'}}{P_{t2}}, \quad \tau_c = \frac{T_{t3c}}{T_{t2}}, \quad \pi_c = \frac{P_{t3c}}{P_{t2}}$$

$$\tau_{Ec} = \frac{T_{t3}}{T_{t3c}}, \quad \pi_{Ec} = \frac{P_{t3}}{P_{t3c}}, \quad \pi_{BL} = \frac{P_{t4L}}{P_{t3}}, \quad \pi_{BH} = \frac{P_{t4}}{P_{t4L}}$$

$$\tau_{Et} = \frac{T_{t5EL}}{T_{t4}}, \quad \pi_{Et} = \frac{P_{t5EL}}{P_{t4}}, \quad \tau_{tH} = \frac{T_{t5BL}}{T_{t4L}}, \quad \pi_{tH} = \frac{P_{t5BL}}{P_{t4L}}$$

$$\tau_{tL} = \frac{T_{t5}}{T_{t5ML}}, \quad \pi_{tL} = \frac{P_{t5}}{P_{t5ML}}, \quad \tau_n = \frac{T_{t9}}{T_{t5}} = 1, \quad \pi_n = \frac{P_{t9}}{P_{t5}}$$

$$\tau_{n'} = \frac{T_{t9'}}{T_{t3'}} = 1, \quad \pi_{n'} = \frac{P_{t9'}}{P_{t3'}}, \quad \tau_{\lambda L} = \frac{c_{p4L}T_{t4L}}{c_{pc}T_o}, \quad \tau_\lambda = \frac{c_{p4}T_{t4}}{c_{pc}T_o}$$

Figure B-1

Flow Stations, Nomenclature, and Components for a Wave Rotor Turbofan Engine

$$\tau_{Ec} = \pi_{Ec}^{\frac{1}{3.5\, e_{EC}}}$$

$$f_{4L} = \frac{\tau_{\lambda L} - \tau_r \tau_o \tau_{Ec}}{\frac{\eta_{BLh}}{.24\, T_o} - \tau_{\lambda L}}$$

$$f_4 = \frac{\left(1 + f_{4L}\right)\left(\tau_\lambda - \tau_{\lambda L}\right)}{\frac{\eta_{BHh}}{.24\, T_o} - \tau_\lambda}$$

$$\pi_{Et} = \frac{1}{\pi_{EC}}$$

$$\pi_{th} = \frac{1}{\pi_{Ec}}$$

$$\tau_{Et} = \pi_{Ec}^{\frac{\left(\gamma_4 - 1\right) e_{Et}}{\gamma_4}}$$

$$\tau_{tH} = \pi_{tH}^{\frac{\left(\gamma_{4L} - 1\right) e_{t4}}{\gamma_{4L}}}$$

$$\epsilon = 1 - \frac{\tau_r \tau_c \left(\tau_{EC} - 1\right)}{\left(1 + f_{4L} + f_4\right) \tau_\lambda \left(1 - \tau_{Et}\right)}$$

$$\tau_{tL} = \frac{\epsilon\left[1+f_{4L}\right]\tau_{\lambda L}+(1-\epsilon)\left[1+f_{4L}+f_4\right]\tau_\lambda\tau_{Et}-\tau_r\left[\tau_c-1+\alpha\left[\tau_{c'}-1\right]\right]}{\epsilon\left[1+f_{4L}\right]\tau_{\lambda L}\tau_{tH}+(1-\epsilon)\left[1+f_{4L}+f_4\right]\tau_\lambda\tau_{Et}}$$

$$\pi_{tL} = \tau_{tL}^{\frac{\gamma_{5ML}}{\left[\gamma_{5ML}-1\right]\ e_{tL}}}$$

$$C = \frac{C_{p5ML}T_{t5ML}}{C_{pc}T_o} = \frac{\epsilon\left[1+f_{4L}\right]\tau_{\lambda L}\tau_{tH}+(1-\epsilon)\left[1+f_{4L}+f_4\right]\tau_\lambda\tau_{Et}}{1+f_{4L}+(1-\epsilon)f_4}$$

$$\frac{P_{t9'}}{P_{9'}} = \frac{P_o}{P_{9'}}\,\pi_r\pi_d\pi_{c'}\pi_{u'}$$

$$\gamma_{9'} = 1.4$$

$$\frac{T_{9'}}{T_o} = \frac{3.5\left[\dfrac{\gamma_{9'}-1}{\gamma_{9'}}\right]\tau_r\tau_{c'}}{\left[\dfrac{P_{t9'}}{P_{9'}}\right]^{\dfrac{\gamma_{9'}-1}{\gamma_{9'}}}}$$

$$M_o\,\frac{U_{9'}}{U_o} = \left\{5\tau_{\lambda AB'}\left[1-\left[\frac{P_{t9'}}{P_{9'}}\right]^{-\left[\frac{\lambda_{9'}-1}{\lambda_{9'}}\right]}\right]\right\}^{1/2}$$

PRIMARY STREAM:

$$\frac{P_{t9}}{P_9} = \frac{P_o}{P_9}\,\pi_r\pi_d\pi_c\pi_{BH}\pi_{BL}\pi_M\pi_{tL}\pi_n$$

$$\gamma_9 = \gamma_{5ML}$$

$$\frac{T_9}{T_o} = \frac{3.5\left[\frac{\gamma_9-1}{\gamma_9}\right]C\tau_{tL}}{\left[\frac{P_{t9}}{P_9}\right]^{\frac{\gamma_9-1}{\gamma_9}}}$$

(NB:'AB and AB' refer to optional use of afterburner)

$$M_o \frac{U_9}{U_o} = \left[5\tau_{\lambda AB}\left[1 - \left[\frac{P_{t9}}{P_9}\right]^{-\left[\frac{\gamma_9-1}{\gamma_9}\right]}\right]\right]^{1/2}$$

$$f_{AB} = \left[1 + f_{4L} + (1-\epsilon)f_4\right] \frac{\tau_{\lambda AB} - C\tau_{tL}}{\frac{\eta_{ABh}}{.24\ T_o} - \tau_{\lambda AB}}$$

$$f_{AB'} = \frac{\tau_{\lambda AB'} - \tau_r\tau_{c'}}{\frac{\eta_{AB'h}}{.24\ T_o} - \tau_{\lambda AB'}}$$

$$A_1 = \left[1 + f_{4L} + (1-\epsilon)\ f_4 + f_{AB}\right]$$

$$X_1 = M_o \frac{U_9}{U_o} + \frac{U_o}{1.4\ M_o U_o} \frac{T_9}{T_o} \left[1 - \frac{P_o}{P_9}\right]$$

$$X_2 = M_o \frac{U_{9'}}{U_o} + \frac{U_o}{1.4\, M_o U_{9'}} \frac{T_{9'}}{T_o} \left[1 - \frac{P_o}{P_{9'}}\right]$$

$$\frac{F}{g_o\left(m_c + m_f\right)} = \frac{a_o}{g_o(1+\alpha)} \left[A_1 X_1 + \alpha\left(1 + f_{AB'}\right) X_2 - (1+\alpha) M_o\right]$$

$$S = \frac{3600\left[f_{4L} + (1-\epsilon) f_4 + f_{AB} + \alpha f_{AB'}\right]}{(1+\alpha)\dfrac{F}{g_o\left(m_c + m_f\right)}}$$

OFF-DESIGN EQUATIONS

Inputs

M_{OR}	α_R	π_{CR}	$\pi_{C'R}$	π_{eCR}
γ_4	γ_{4L}	γ_{5ML}	$\tau_{\lambda R}$	$\tau_{\lambda LR}$
e_C	$e_{C'}$	e_{ec}	e_{tH}	e_{et}
e_{tL}	ϵ_R	τ_{tLR}	τ_λ	$\tau_{\lambda L}$
M_O				

Equations

$$\tau_{tL} = \tau_{tLR}$$

$$\tau_{CR} = \pi_{CR}^{\frac{1}{3.5\, e_C}}$$

$$\tau_{C'R} = \pi_{C'R}^{\frac{1}{3.5\, e_{C'}}}$$

$$\tau_{eCR} = \pi_{eCR}^{\frac{1}{3.5\, e_{ec}}}$$

$$\tau_{tHR} = \left[\pi_{eCR}\right]^{\frac{1-\gamma_{4L}}{\gamma_{4L}} e_{tH}} \quad \text{since } \pi_{th} = 1/\pi_{ec}$$

$$\tau_{etR} = \left[\pi_{eCR}\right]^{\frac{1-\gamma_{4}}{\gamma_{4}} e_{et}} \quad \text{since } \pi_{et} = 1/\pi_{eC}$$

$$\eta_{th} = \frac{1 - \tau_{thR}}{1 - \left[\tau_{thR}\right]^{\frac{1}{e_{th}}}}$$

$$\eta_{tL} = \frac{1 - \tau_{tLR}}{1 - \left[\tau_{tLR}\right]^{\frac{1}{e_{tL}}}}$$

$$\eta_{et} = \frac{1 - \tau_{etR}}{1 - \left[\tau_{etR}\right]^{\frac{1}{e_{et}}}}$$

$$\tau_{HZ} = 1 + \frac{M_{O_R}^2}{5}$$

$$\eta_C = \frac{\pi_{CR}^{\frac{1}{3.5}} - 1}{\tau_{CR} - 1}$$

$$\eta_{C'} = \frac{\pi_{C'R}^{\frac{1}{3.5}} - 1}{\tau_{C'R} - 1}$$

$$\eta_{eC} = \frac{\pi_{eCR}^{\frac{1}{3.5}} - 1}{\tau_{eCR} - 1}$$

$$\tau_r = 1 + \frac{M_o^2}{5}$$

$$f_{4LR} = \frac{\tau_{\lambda LR} - \tau_{rR}\tau_{CR}\tau_{eCR}}{\frac{\eta_{BL} h}{0.24\ T_o} - \tau_{\lambda LR}}$$

$$f_{4R} = \frac{\left(1 + f_{4LR}\right)\left(\tau_{\lambda R} - \tau_{\lambda LR}\right)}{\frac{\eta_{BH} h}{0.24\ T_o} - \tau_{\lambda R}}$$

$$\epsilon_R = 1 - \frac{\tau_{rR}\tau_{CR}\left(\tau_{ecR} - 1\right)}{\left(1 + f_{4LR} + f_{4R}\right)\ \tau_{\lambda R}\ \left(1 - \tau_{etR}\right)}$$

$$C_R = \frac{\epsilon_R\ \left(1 + f_{4LR}\right)\ \tau_{\lambda LR}\ \tau_{tHR} + \left(1 - \epsilon_R\right)\left(1 + f_{4LR} + f_{4R}\right)\ \tau_{\lambda R}\ \tau_{etR}}{1 + f_{4LR} + \left(1 - \epsilon_R\right)\ f_{4R}}$$

$$\pi_{ec} = \pi_{ecR}\ \frac{\epsilon}{\epsilon_R}\left[\frac{1 + f_{4L}}{1 + f_{4LR}}\ \frac{1 + f_{4LR} + \left(1 - \epsilon_R\right)\ f_{4R}}{1 + f_{4L} + (1 - \epsilon)\ f_4}\right]\sqrt{\frac{C_R\tau_{\lambda L}}{C\tau_{\lambda LR}}}$$

$$\pi_{th} = \frac{1}{\pi_{ec}}$$

$$\pi_{et} = \pi_{th}$$

$$\tau_{th} = 1 - \eta_{th}\left[1 - \pi_{th}^{\frac{\gamma_{4L} - 1}{\gamma_{4L}}}\right]$$

$$\tau_{et} = 1 - \eta_{et}\left[1 - \pi_{et}^{\frac{\gamma_4 - 1}{\gamma_4}}\right]$$

$$C = \frac{\epsilon\left(1 + f_{4L}\right)\tau_{\lambda L}\tau_{th} + (1 - \epsilon)\left(1 + f_4 + f_{4L}\right)\tau_{\lambda}\tau_{et}}{1 + f_{4L} + (1 - \epsilon)\ f_4}$$

$$\tau_{ec} = \frac{1 + \left[\pi_{ec}^{\frac{1}{3.5}} - 1\right]}{\eta_{ec}}$$

$$\tau_c = \frac{(1 - \epsilon)\, \tau_\lambda \left[1 - \tau_{et}\right]}{\tau_r\left[\tau_{ec} - 1\right]} \left[1 + f_4 + f_{4L}\right]$$

$$\pi_c = \left[1 + \eta_c \left(\tau_c - 1\right)\right]^{3.5}$$

$$\pi_{c'} = \left[1 + \eta_{c'} \left(\tau_{c'} - 1\right)\right]^{3.5}$$

$$\alpha = \alpha_R \frac{\pi_{C'}}{\pi_{C'R}} \frac{\pi_{CR}}{\pi_C} \left[\frac{\tau_{rR}\tau_{C'R}}{\tau_r\tau_{C'}} \frac{C}{C_R}\right]^{\frac{1}{2}} \frac{1 + f_{4L} + (1 - \epsilon)\, f_4}{1 + f_{4LR} + \left[1 - \epsilon_R\right] f_{4R}}$$

$$t_{C'} = 1 + \left[\tau_{C'R} - 1\right] \frac{1 + \alpha_R}{1 + \alpha} \frac{\tau_{rR}}{\tau_r} \frac{C}{C_R} \frac{1 + f_{4L} + (1 - \epsilon)\, f_4}{1 + f_{4LR} + \left[1 - \epsilon_R\right] f_{4R}}$$

Let $m = \left[1 + f_4 + f_{4L}\right] \tau_\lambda \tau_{et} \left[1 - \tau_{tL}\right]$

$$\epsilon = \frac{\tau_r \left[\tau_c - 1 + \alpha\left[\tau_{c'} - 1\right]\right] - m}{\left[1 + f_{4L}\right] \tau_{\lambda L} \left[1 - \tau_{tL}\tau_{th}\right] - m}$$

$$f_{4L} = \frac{\tau_{\lambda L} - \tau_r \tau_C \tau_{eC}}{\frac{\eta_{BL} h}{0.24\ T_o} - \tau_{\lambda L}}$$

$$f_4 = \frac{\left[1 + f_{4L}\right]\left[\tau_\lambda - \tau_{\lambda L}\right]}{\frac{\eta_{BH} h}{0.24\ T_o} - \tau_\lambda}$$

Appendix C

OPTIMUM BYPASS EQUATIONS

The first 13 equations are the same as the first 13 on-design equations in Appendix B.

$$\pi = \left[\pi_r \pi_d \pi_c \pi_{ec} \pi_{BH} \pi_{BL} \pi_M \pi_{et} \pi_n\right]^{\frac{\gamma_9 - 1}{\gamma_9}}$$

$$C = \frac{\epsilon\left[1 + f_{4L}\right]\tau_{\lambda L}\tau_{tH} + (1 - \epsilon)\left[1 + f_{4L} + f_4\right]\tau_\lambda \tau_{Et}}{1 + f_{4L} + (1 - \epsilon)f_4}$$

$$M_o \frac{U_{9'}}{U_o} = \left\{5\ \tau_r \tau_{c'} \left[1 - \left(\pi_r \pi_d \pi_{c'} \pi_{n'}\right)^{-\frac{1}{3.5}}\right]\right\}^{1/2}$$

$$\tau_{t\ell o} = \frac{1}{\pi} + \frac{5}{4C}\left[\frac{\tau_r\left(\tau_{c'} - 1\right)}{M_o \frac{U_{9'}}{U_o} - M_o}\right]^2$$

$$\text{Let } G = \left[\frac{\tau_r\left(\tau_{c'} - 1\right)}{M_o \frac{U_{9'}}{U_o} - M_o}\left\{1 + \frac{1 - e_{t\ell}}{e_{t\ell}}\frac{1}{\pi}\ \tau_{t\ell}^{\ -\frac{1}{e_{t\ell}}}\right\}\right]^2$$

Iterate on next equation to get τ_{tL}. Start with $\tau_{t\ell o}$.

$$\tau_{t\ell} = \frac{1}{\pi}\,\tau_{t\ell}^{\;-\left[\frac{1 - e_{t\ell}}{e_{t\ell}}\right]} + \frac{5G^2}{4C}$$

$$B_1 = (1-\epsilon)\left[1 + f_{4L} + f_4\right]\,\tau_\lambda \tau_{et}$$

$$B_2 = B_1\left[1 - \tau_{tL}\right] + \epsilon\left[1 + f_{4L}\right]\,\tau_{\lambda L}\,\left[1 - \tau_{th}\tau_{tL}\right] - \left[\tau_c - 1\right]\tau_r$$

$$\alpha = \frac{B_2}{\tau_r\left[\tau_{c'} - 1\right]}$$

$$\pi_{tL} = \tau_{t\ell}^{\;\frac{\gamma_9}{\left[\gamma_9 - 1\right] e_{t\ell}}}$$

$$\frac{P_{t9}}{P_9} = \pi^{\frac{\gamma_9}{\gamma_9 - 1}}\,\pi_{t\ell}$$

$$M_o\,\frac{U_9}{U_o} = \left[5\;C\;\tau_{tL}\left[1 - \left[\frac{P_{t9}}{P_9}\right]^{-\left[\frac{\gamma_9 - 1}{\gamma_9}\right]}\right]\right]^{1/2}$$

$$SPTH = \frac{a_o}{(1+\alpha)\ 32.17}\left[\left[1+f_{4L}+(1-\epsilon)f_4\right] M_o \frac{U_9}{U_o} - M_o + \alpha\left\{M_o \frac{U_{9'}}{U_o} - M_o\right\}\right]$$

$$SFC = 3600\ \frac{f_{4L} + (1 - \epsilon)\ f_4}{(1 + \alpha)\ SPTH}$$

Appendix D

PARTIALLY OPEN TUBE EQUATIONS

(Refer to Figure D-1 for geometry and nomenclature)

Equations for Partially Open Inflow:

$$\rho_m u_m A_m = \rho_{th} u_{th} A_{th} = \rho_w u_w A_w, \text{ continuity}$$

$$\left[\frac{\gamma-1}{2}\right] v_m{}^2 + a_m{}^2 = \left[\frac{\gamma-1}{2}\right] u_w{}^2 + a_w{}^2 = a_{or}{}^2, \text{ energy}$$

where $v_m = \left[u_m \cos\theta,\ u_m \sin\theta - V_T\right]$

$$P_m\left[1 + \frac{\gamma-1}{2}\left[\frac{V_m}{a_m}\right]^2\right]^{\frac{\gamma}{\gamma-1}} = P_{th}\left[1 + \frac{\gamma-1}{2}\left[\frac{u_{th}}{a_{th}}\right]^2 (1+L)\right]^{\frac{\gamma}{\gamma-1}}, \text{ momentum}$$

where L = loss term due to impulsive blade loading

$$\frac{\gamma-1}{2} QQ = \left[\frac{\gamma-1}{2}\right] u_w + a_w\left[\frac{S_Q}{S_w}\right], \text{ Riemann Invariant}$$

$$p_w = \frac{a_w{}^2 \rho_w}{\gamma},\ p_m = \frac{a_m{}^2 \rho_m}{\gamma}, \text{ Ideal Gas Equation of State}$$

where $S_w = a_w p_w^{\frac{1-\gamma}{2\gamma}},\quad S_m = a_{om} p_{om}^{\frac{1-\gamma}{2\gamma}}$

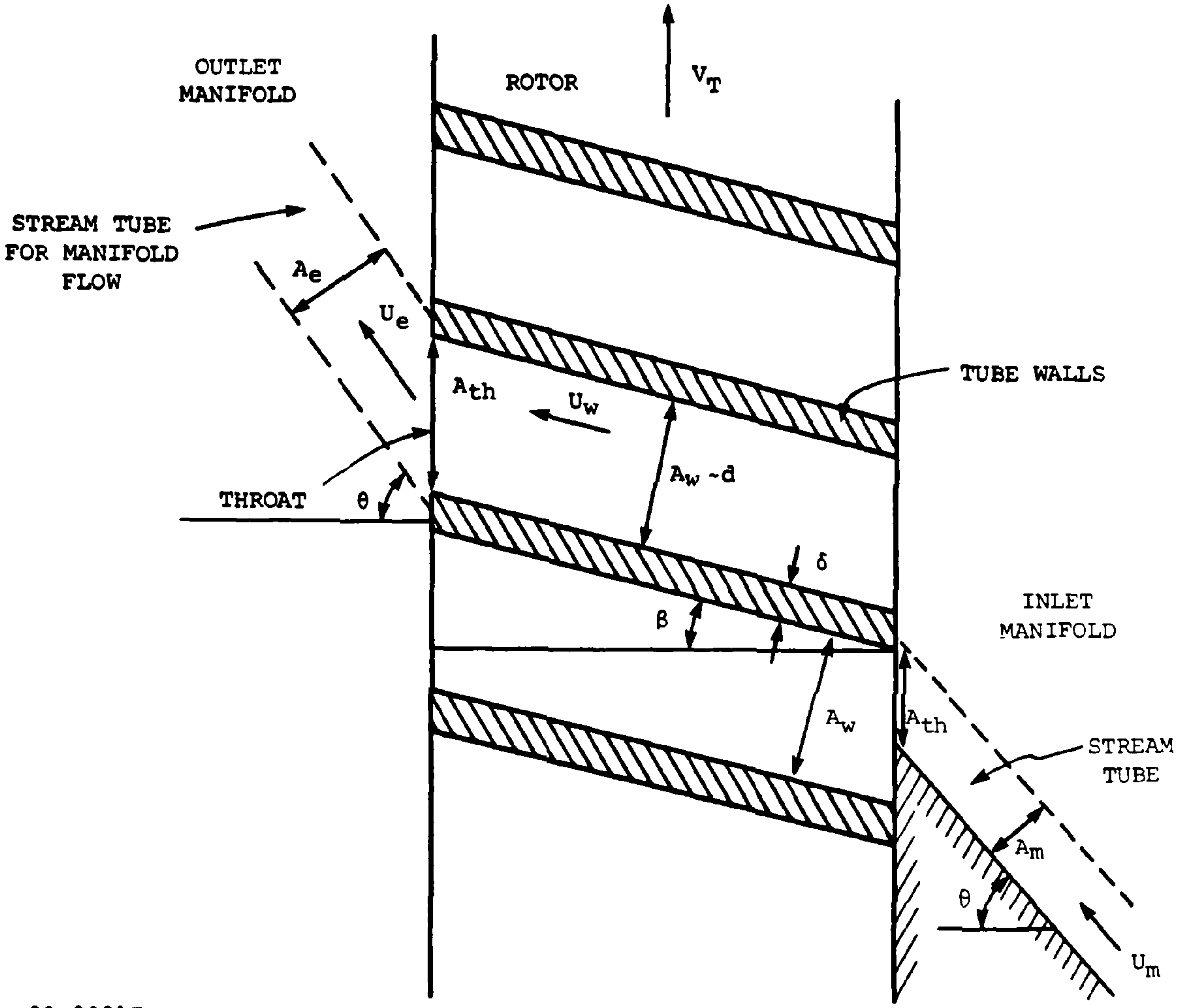


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Figure D-1

Inflow and Outflow Conditions showing rotor tube geometry and stream tube for flow in the manifolds.

Subsonic Throat:

$p_{th} = p_w$, assumes loss of head

Sonic Throat:

$u_{th} = a_{th}$

Equations for Partially Open Outflow:

Refer to text Equations 4-4 to 4-9 with $\Phi < 1$.

Appendix E

WAVE DIAGRAM EQUATIONS

INPUTS (Refer to Figure E-1 for nomenclature)

a_{3D} u_{1d} a_{1d}

ρ_{1d} ω_d ω_D

EQUATIONS

Define:

$u_R = u_{1d}/a_{1d}$ and $X = 0.5(\gamma+1)\ u_R$

$$M = 0.5\ [X + (X^2 + 4)^{1/2}]$$

$$a_R = \left[1 + \frac{2(\gamma-1)(\gamma M^2+1)(M^2-1)}{(\gamma+1)^2\ M^2}\right]^{1/2}$$

$$K = \frac{a_{3D}}{a_{1d}a_R{}^2}$$

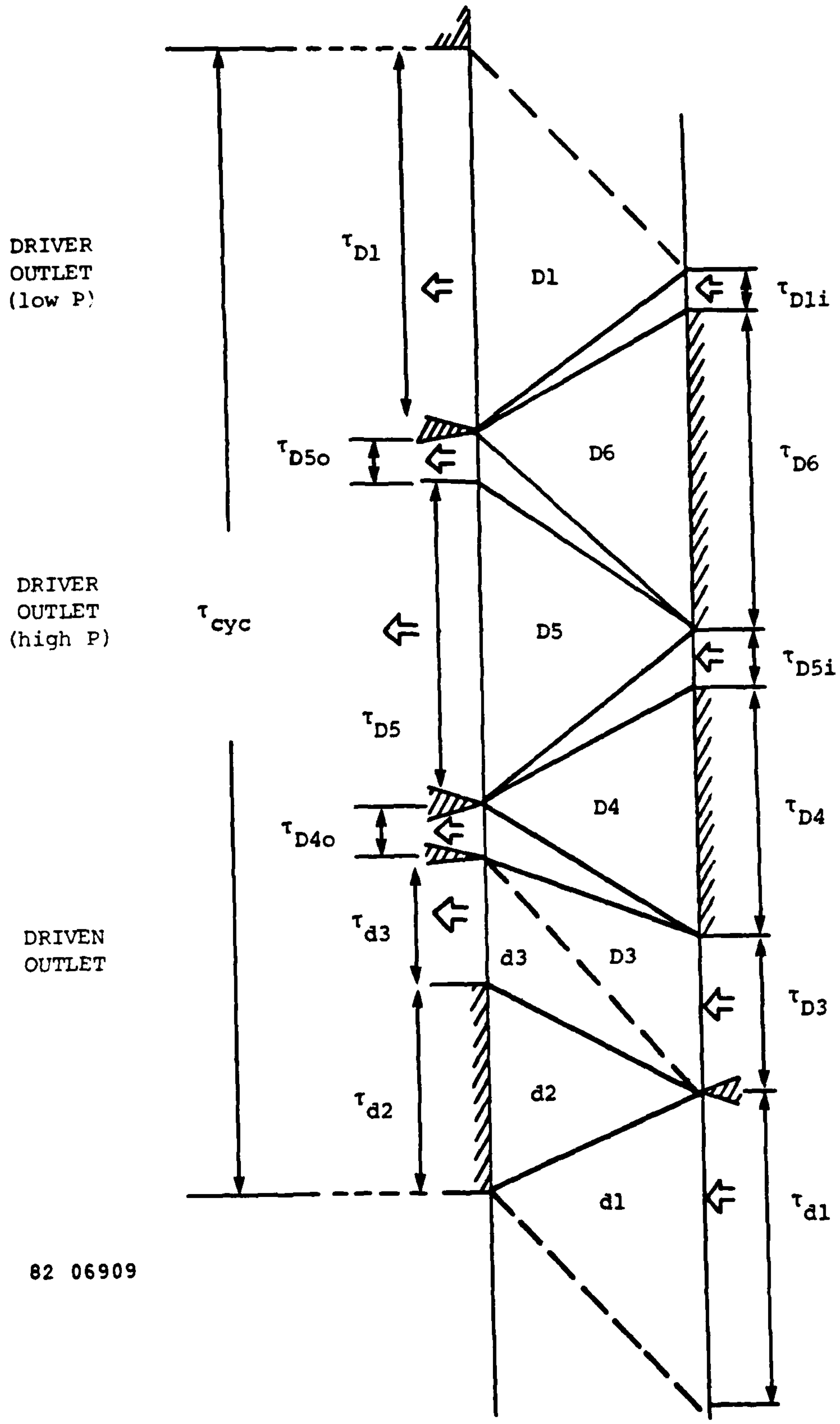

Figure E-1

Ideal Wave Diagram for a Nine Port Pressure Exchanger Wave Rotor portraying the cycle time τ_{cyc}, identification of uniform flow regions, port and endwall times.

Driven Inlet:

$$p_{1d} = \frac{\rho_{1d} a_{1d}^2}{\gamma}, \qquad T_{1d} = \frac{a_{1d}^2 \omega_d}{\gamma R}$$

Stagnant Driven:

$$u_{2d} = 0, \qquad a_{2d} = a_R a_{1d}$$

$$p_{2d} = p_{1d}\left[1 + \frac{2\gamma(M^2-1)}{\gamma + 1}\right]$$

$$\rho_{2d} = \rho_{1d}\left[\frac{(\gamma+1)M_{1d}^2}{(\gamma-1)M_{1d}^2+2}\right]$$

$$T_{2d} = \frac{a_{2d}^2 \omega_d}{\gamma R}$$

Driven Outlet:

$$u_{3d} = u_R a_{2d}, \qquad a_{3d} = a_R a_{2d}$$

$$p_{3d} = p_{2d}\left[\frac{p_{2d}}{p_{1d}}\right], \qquad \rho_{3d} = \rho_{2d}\left[\frac{\rho_{2d}}{\rho_{1d}}\right]$$

$$T_{3d} = \frac{a_{3d}^2 \omega_d}{\gamma R}$$

Driver Inlet:

$$p_{3D} = p_{3d} , \quad \rho_{3D} = \frac{p_{3D}\gamma}{a_{3D}^2} , \quad T_{3D} = \frac{a_{3D}^2 \omega_D}{\gamma R}$$

$$u_{3D} = u_R a_R a_{1d}$$

Stagnant Driver (high pressure):

$$u_{4D} = 0, \quad a_{4D} = a_{3D}\left[1 - \frac{(\gamma - 1)u_R}{2a_R K}\right]$$

$$\rho_{4D} = \rho_{3D}\left[\frac{a_{4D}}{a_{3D}}\right]^{\frac{2}{\gamma-1}} , \quad p_{4D} = \frac{\rho_{4D} a_{4D}^2}{\gamma}$$

$$T_{4D} = \frac{a_{4D}^2 \omega_D}{\gamma R}$$

Driver Outlet (low pressure):

$$u_{1D} = u_{1d}, \quad p_{1D} = p_{1d}$$

$$T_{1D} = T_{3D}\left[\frac{P_{1D}}{P_{3D}}\right]^{\frac{\gamma-1}{\gamma}}, \quad \rho_{1D} = \frac{P_{1D}\omega_D}{RT_{1D}}$$

$$a_{1D} = \left[\frac{\gamma RT_{1D}}{\omega_D}\right]^{1/2}, \quad K' = \frac{a_{1D}}{a_{1d}}$$

Stagnant Driver (low pressure):

$$u_{6D} = 0, \; a_{6D} = a_{1d}K'\left[1 + \frac{(\gamma-1)u_R}{2K'}\right]$$

$$\rho_{6D} = \rho_{1D}\left[\frac{a_{6D}}{a_{1D}}\right]^{\frac{2}{\gamma-1}}, \quad T_{6D} = \frac{a_{6D}{}^2\omega_D}{\gamma R}$$

$$P_{6D} = \frac{\rho_{6D}a_{6D}{}^2}{\gamma}$$

Driver Outlet (high pressure):

$$P_5 = \frac{2a_{1D}}{\gamma-1} + u_{1D}$$

$$Q_5 = \frac{2a_{1D}}{\gamma-1} - u_{3D}$$

$$u_{5D} = 0.5\left(P_5 - Q_5\right), \quad a_{5D} = \frac{\gamma-1}{4}\left(P_5 + Q_5\right)$$

$$\rho_{5D} = \rho_{6D}\left[\frac{a_{5D}}{a_{6D}}\right]^{\frac{2}{\gamma-1}}, \quad T_{5D} = \frac{a_{5D}{}^2\omega_D}{\gamma R}$$

$$P_{5D} = \frac{\rho_{5D}a_{5D}{}^2}{\gamma}$$

END

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